

ANALYSIS OF A COMPLEX OF STATISTICAL VARIABLES INTO PRINCIPAL COMPONENTS¹

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1. INTRODUCTION

Consider n variables attaching to each individual of a population. These statistical variables x_1, x_2, \dots, x_n might for example be scores made by school children in tests of speed and skill in solving arithmetical problems or in reading; or they might be various physical properties of telephone poles, or the rates of exchange among various currencies. The x 's will ordinarily be correlated. It is natural to ask whether some more fundamental set of independent variables exists, perhaps fewer in number than the x 's, which determine the values the x 's will take. If $\gamma_1, \gamma_2, \dots$ are such variables, we shall